1.1 Real Numbers and Number Operations

**Goal**
- Graph, order, and use real numbers.

**Your Notes**

**VOCABULARY**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin</td>
<td>The point labeled 0 on a real number line</td>
</tr>
<tr>
<td>Graph</td>
<td>The point on a number line that corresponds to a real number</td>
</tr>
<tr>
<td>Coordinate</td>
<td>The number that corresponds to a point on a number line</td>
</tr>
<tr>
<td>Opposite</td>
<td>The opposite, or additive inverse, of any number ( a ) is (-a).</td>
</tr>
<tr>
<td>Reciprocal</td>
<td>The reciprocal, or multiplicative inverse, of any nonzero number ( a ) is ( \frac{1}{a} ).</td>
</tr>
</tbody>
</table>

**REAL NUMBERS**

The real numbers consist of the **rational** numbers and the **irrational** numbers. Two subsets of the rational numbers are the **whole numbers** (0, 1, 2, ...) and the **integers** (..., -3, -2, -1, 0, 1, 2, 3, ...).

**Rational Numbers**
- **Can** be written as the ratios of integers
- Can be written as **decimals** that terminate or repeat

**Irrational Numbers**
- **Cannot** be written as the ratios of integers
- Cannot be written as decimals that **terminate** or **repeat**
Example 1  Graph Real Numbers on a Number Line

Graph the real numbers \(-\frac{13}{5}, 1.4, \) and \(\sqrt{6}\) on a number line.

Solution

Note that \(-\frac{13}{5} = -2.6\). Use a calculator to approximate \(\sqrt{6}\) to the nearest tenth: \(\sqrt{6} \approx 2.4\). So, graph \(-\frac{13}{5}\) between \(-3\) and \(-2\), graph 1.4 between 1 and 2, and graph \(\sqrt{6}\) between 2 and 3.

Example 2  Compare Real Numbers

Compare the numbers \(-3\) and \(2\) using the symbol < or >.

Solution

Graph both numbers on a real number line.

Because \(-3\) is to the left of 2, it follows that \(-3\) is less than 2, which is written using symbols as \(-3 < 2\).

Checkpoint  Complete the following exercises.

1. Graph the numbers \(\sqrt{5}, -2, -\frac{1}{2}\), and 3.2.

2. Write the numbers above in order from least to greatest.

\(-2, -\frac{1}{2}, \sqrt{5}, 3.2\)
Properties of Addition and Multiplication

Let \( a, b, \) and \( c \) be real numbers.

<table>
<thead>
<tr>
<th>Property</th>
<th>Addition</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Commutative</td>
<td>( a + b = b + a )</td>
<td>( ab = ba )</td>
</tr>
<tr>
<td>Associative</td>
<td>((a + b) + c = a + (b + c))</td>
<td>((ab)c = a(bc))</td>
</tr>
<tr>
<td>Identity</td>
<td>( a + 0 = a ), ( 0 + a = a )</td>
<td>( a \cdot 1 = a ), ( 1 \cdot a = a )</td>
</tr>
<tr>
<td>Inverse</td>
<td>( a + (\bar{a}) = 0 )</td>
<td>( a \cdot \frac{1}{a} = 1, a \neq 0 )</td>
</tr>
</tbody>
</table>

The following property involves both addition and multiplication.

Distributive \( a(b + c) = \_ab + \_ac \)

Example 3 Use Properties of Real Numbers

Use the distributive property to evaluate \( 7 \cdot 302 \).

\[
7 \cdot 302 = 7(300 + 2) \quad \text{Rewrite } 302 \text{ as } 300 + 2. \\
= 7(300) + 7(2) \quad \text{Distributive property} \\
= 2100 + 14 \quad \text{Multiply.} \\
= 2114 \quad \text{Simplify.}
\]

Example 4 Operations with Real Numbers

a. Find the difference of \(-11\) and 3.

\[
-11 - 3 = -11 + (-3) \quad \text{Add } -3, \text{ the opposite of } 3. \\
= -14 \quad \text{Simplify.}
\]

b. Find the quotient of \(-20\) and \(\frac{1}{5}\).

\[
-20 \div \frac{1}{5} = -20 \cdot 5 \quad \text{Multiply by } 5, \text{ the reciprocal of } \frac{1}{5}. \\
= -100 \quad \text{Simplify.}
\]

Homework
1.2 Algebraic Expressions and Models

Goal • Define and use algebraic expressions.

Your Notes

VOCABULARY

Exponent The number or variable that represents the number of times the base of a power is used as a factor

Power An expression that represents repeated multiplication of the same factor

Base In a power, the number or expression used as a factor in a repeated multiplication

Numerical expression An expression that consists of numbers, operations, and grouping symbols

Algebraic expression An expression involving variables

Variable A letter used to represent one or more numbers

Example 1 Evaluate Powers

a. \((-3)^2 = (-3) \cdot (-3) = 9\)
b. \(-3^2 = -(3 \cdot 3) = -9\)

Checkpoint Evaluate the power.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (5^4)</td>
<td>2. (-5^4)</td>
</tr>
<tr>
<td>625</td>
<td>-625</td>
</tr>
<tr>
<td>3. ((-5)^4)</td>
<td>4. (5^1)</td>
</tr>
<tr>
<td>625</td>
<td>5</td>
</tr>
</tbody>
</table>

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ORDER OF OPERATIONS

Step 1 Perform operations that occur within grouping symbols.
Step 2 Evaluate powers.
Step 3 Perform multiplications and divisions from left to right.
Step 4 Perform additions and subtractions from left to right.

Example 2  Use Order of Operations

\[
12 \div 6 - 4 \times 2^2 = 12 \div 6 - 4 \times 4
\]
Evaluate power.
\[
= 2 - 4 \times 4
\]
Divide.
\[
= 2 - 16
\]
Multiply.
\[
= -14
\]
Subtract.

Example 3  Evaluate an Algebraic Expression

a. Evaluate \(3(x - 1) + 5x\) when \(x = 2\).

\[
3(x - 1) + 5x
\]
\[
= 3(2 - 1) + 5(2)
\]
Substitute 2 for \(x\).
\[
= 3(1) + 5(2)
\]
Subtract within parentheses.
\[
= 3 + 10
\]
Multiply.
\[
= 13
\]
Add.

b. Evaluate \(-6y^2 - 11y + 34\) when \(y = -5\).

\[
-6y^2 - 11y + 34
\]
\[
= -6(-5)^2 - 11(-5) + 34
\]
Substitute \(-5\) for \(y\).
\[
= -6(25) - 11(-5) + 34
\]
Evaluate power.
\[
= -150 + 55 + 34
\]
Multiply.
\[
= -61
\]
Add.
Checkpoint Complete the following exercises.

5. Evaluate
   \[ 8 + 3(7 - 5)^2. \]
   \[ 20 \]

6. Evaluate \( 2y^2(y - 1) \) when \( y = 3 \).
   \[ 36 \]

Example 4 Write and Evaluate a Mathematical Model

Recreation You have $120 and are buying amusement park tickets that cost $26 each. Write an expression that shows how much money you have left after buying \( t \) tickets. Evaluate the expression for \( t = 2 \) and \( t = 4 \).

Solution

Verbal Model

<table>
<thead>
<tr>
<th>Original amount</th>
<th>Price per ticket</th>
<th>Number of tickets bought</th>
</tr>
</thead>
</table>

Labels

- Original amount = 120 (dollars)
- Price per ticket = 26 (dollars per ticket)
- Number of tickets bought = \( t \) (tickets)

Algebraic Model

\[ 120 - 26t \]

When you buy 2 tickets, you have \( 120 - 26(2) = 68 \) left.

When you buy 4 tickets, you have \( 120 - 26(4) = 16 \) left.

Checkpoint Complete the following exercise.

7. In Example 4, write an expression that shows how much money you have left after buying \( t \) tickets if you have $180 and the tickets cost $39 each. Then evaluate the expression when \( t = 2 \) and \( t = 4 \).

\[ 180 - 39t; \$102; \$24 \]
Simplifying Algebraic Expressions

Goal

• Simplify algebraic expressions.

VOCABULARY

Terms  The parts of an algebraic expression that are added together

Coefficient  When a term is the product of a number and a power of a variable, the number is the coefficient.

Like terms  Terms that have the same variable parts

Constant term  A term that has a number part but no variable part (Constants terms are like terms.)

Simplified expression  An expression in which all grouping symbols have been removed and all like terms have been combined

Example 1  Identify Terms in an Expression

Identify the terms in the expression 6 – 5x + x².

Write the expression as a sum.
6 – 5x + x² = 6 + (–5x) + x²

The terms of the expression are 6, –5x, and x².

Example 2  Identify Coefficients and Like Terms

Identify the coefficients and like terms in the expression 4x² – 6x + 3 + 8x – 5.

Write the expression as a sum.
4x² – 6x + 3 + 8x – 5 = 4x² + (–6x) + 3 + 8x + (–5)

The coefficients of the expression are 4, –6, and 8.
The terms –6x and 8x are like terms. The terms 3 and –5 are also like terms.
**Checkpoint** Complete the following exercise.

1. Identify the terms, coefficients, and like terms in the expression $2x^3 - 6x^2 + 4 + 5x^2 - 2x - 4x^3$.

   terms: $2x^3, -6x^2, 4, 5x^2, -2x, -4x^3$;
   coefficients: $2, -6, 5, -2, -4$;
   like terms: $2x^3$ and $-4x^3$, $-6x^2$ and $5x^2$

**Example 3**  
**Simplify by Combining Like Terms**

Simplify the expression.

a. $11x - 5x$

Solution

$a. \quad 11x - 5x = (\underline{11} - \underline{5})x$  
Distributive property  
$= \underline{6}x$  
Subtract.

b. $14x - 6y + 5x + 13y$

Solution

$b. \quad 14x - 6y + 5x + 13y$  
$= (\underline{14}x + 5x) + (-6y + \underline{13}y)$  
Group like terms.  
$= (\underline{14} + \underline{5})x + (-\underline{6} + \underline{13})y$  
Distributive property  
$= \underline{19}x + \underline{7}y$  
Add.

**Example 4**  
**Simplify Expressions with Grouping Symbols**

Simplify the expression $2(y + 5) - 3(y - 9)$.

Solution

$2(y + 5) - 3(y - 9)$  
$= 2y + 10 - 3y + \underline{27}$  
Distributive property  
$= (\underline{2y} - 3y) + (\underline{10} + \underline{27})$  
Group like terms.  
$= (\underline{2} - \underline{3})y + (10 + 27)$  
Distributive property  
$= \underline{-y} + 37$  
Add or subtract.

**Homework**

**Checkpoint** Simplify the expression.

2. $11n - 6 + 14 - 2n$

   $9n + 8$

3. $6(x - 4) - 3(x - 11)$

   $3x + 9$
1.4 Solving Linear Equations

Goal • Solve linear equations.

Your Notes

VOCABULARY

Equation A statement in which two expressions are equal

Linear equation in one variable An equation that can be written in the form $ax = b$ where $a$ and $b$ are constants and $a \neq 0$

Solution of an equation A number that produces a true statement when substituted for the variable in the equation

PROPERTIES OF EQUALITY

Addition Add the same number to each side:
Property If $a = b$, then $a + c = b + c$.

Subtraction Subtract the same number from each side:
Property If $a = b$, then $a - c = b - c$.

Multiplication Multiply each side by the same nonzero number:
Property If $a = b$ and $c \neq 0$, then $ac = bc$.

Division Divide each side by the same nonzero number:
Property If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.

Example 1 Solve a One-Step Equation

Solve $x + 3 = 7$.

Solution

$x + 3 = 7$ Write original equation.

$x + 3 - 3 = 7 - 3$ Subtract 3 from each side.

$x = 4$ Simplify.
Example 2  Solve a Multi-Step Equation

Solve $-2x - 4 = -18$.

Solution

$-2x - 4 = -18$  Write original equation.

$-2x = -14$  Add 4 to each side to isolate the variable.

$x = 7$  Divide each side by $-2$.

Example 3  Solve an Equation with Variables on Both Sides

Solve $2y + 4 = 5y - 2$.

Solution

$2y + 4 = 5y - 2$  Write original equation.

$4 = 3y - 2$  Subtract 2y from each side.

$6 = 3y$  Add 2 to each side.

$2 = y$  Divide each side by 3.

Checkpoint  Solve the equation. Check your solution.

1. $x - 5 = -2$

3

2. $15a - 6 = 9$

1

3. $x - 4 = 3x + 14$

-9
Example 4  Use the Distributive Property

Solve $3(4x - 3) = 7(4x + 3) - 31x$.

Write original equation.

$3(4x - 3) = 7(4x + 3) - 31x$

Distributive property

$12x - 9 = 28x + 21 - 31x$

Combine like terms.

$12x - 9 = -3x + 21$

Add $3x$ to each side.

$15x - 9 = 21$

Add $9$ to each side.

$15x = 30$

Divide each side by $15$.

$x = 2$

Example 5  Solve an Equation with Fractions

Solve $\frac{2}{3}y + \frac{1}{2} = y - \frac{3}{4}$.

Write original equation.

$\frac{2}{3}y + \frac{1}{2} = y - \frac{3}{4}$

Multiply each side by the LCD, $12$.

$12\left(\frac{2}{3}y + \frac{1}{2}\right) = 12\left(y - \frac{3}{4}\right)$

Distributive property

$8y + 6 = 12y - 9$

Subtract $8y$ from each side.

$6 = 4y - 9$

Add $9$ to each side.

$15 = 4y$

Divide each side by $4$.

$\frac{15}{4} = y$

Checkpoint  Solve the equation. Check your solution.

4. $3(x + 3) = -2(x - 7) - 5x$

5. $3 - \frac{1}{3}x = \frac{7}{12}x + \frac{2}{3}$
1.5 Rewriting Equations and Formulas

Goal • Rewrite common formulas and equations that have more than one variable.

Your Notes

COMMON FORMULAS

<table>
<thead>
<tr>
<th>FORMULA</th>
<th>VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>( d = \frac{1}{2}rt )</td>
</tr>
<tr>
<td>Simple Interest</td>
<td>( I = Prt )</td>
</tr>
<tr>
<td>Temperature</td>
<td>( F = \frac{9}{5}C + 32 )</td>
</tr>
</tbody>
</table>

Temperature Conversion

\[ F = \frac{9}{5}C + 32 \]
F = degrees Fahrenheit,
C = degrees Celsius

GEOMETRY FORMULAS

RECTANGLE

Perimeter \( P = 2l + 2w \)
Area \( A = lw \)

TRIANGLE

Perimeter \( P = a + b + c \)
Area \( A = \frac{1}{2}bh \)

TRAPEZOID

Area \( A = \frac{1}{2}(b_1 + b_2)h \)

CIRCLE

Circumference \( C = \pi d \) or \( C = 2\pi r \)
Area \( A = \pi r^2 \)

Example 1 Rewrite a Common Formula

Solve \( C = 2\pi r \) for \( r \).

Solution

\[ \frac{C}{2\pi} = r \]  
Write original formula.

Divide each side by \( 2\pi \).
Example 2  Use a Rewritten Formula

Find the radius of a circular pool with a circumference of 26 meters.

\[ r = \frac{C}{2\pi} \]

Use the rewritten formula from Example 1.

\[ r = \frac{26}{2\pi} \]

Substitute 26 for C.

\[ = \frac{13}{\pi} \approx 4.14 \]

Divide. Use the π key on a calculator.

The radius of the pool is about 4.14 meters.

Example 3  Calculate the Value of a Variable

Find the value of y in the equation \(3x + 4y = 8\) when \(x = -4\) and when \(x = 12\).

Solution

Method 1: First substitute for x. Then solve for y.

When \(x = -4\):

\[3(-4) + 4y = 8\]

\[-12 + 4y = 8\]

\[4y = 20\]

\[y = 5\]

When \(x = 12\):

\[3(12) + 4y = 8\]

\[36 + 4y = 8\]

\[4y = -28\]

\[y = -7\]

Method 2: First solve for y. Then substitute for x.

\[3x + 4y = 8\]

Write original equation.

\[4y = \frac{-3x}{4} + 8\]

Subtract \(\frac{3x}{4}\) from each side.

\[y = \frac{-3}{4}x + 2\]

Divide each side by \(\frac{4}{4}\).

When \(x = -4\):

\[y = \frac{-3}{4}(-4) + 2 = 5\]

When \(x = 12\):

\[y = \frac{-3}{4}(12) + 2 = -7\]
Checkpoint Complete the following exercises.

1. Solve the formula \( P = 2l + 2w \) for \( w \). Then find the width of a rectangle with length 9 meters and perimeter 32 meters.

\[
\begin{align*}
w &= \frac{P - 2l}{2} \\
    &= \frac{1}{2}P - \frac{1}{2}l; 7 \text{ meters}
\end{align*}
\]

2. Find the value of \( y \) in the equation \( 5x - 4y = 9 \) when \( x = -3 \) and \( x = 9 \).

\[
-6; 9
\]

Example 4 Use an Equation with Two Variables

Postage Sam is buying some 41-cent stamps and some 58-cent stamps. Write an equation with more than one variable to represent the total cost of the stamps. What is the total cost of twelve 41-cent stamps and ten 58-cent stamps?

Solution

<table>
<thead>
<tr>
<th>Verbal Model</th>
<th>Total cost = Cost ( \cdot ) Number of 41¢ stamps + Cost ( \cdot ) Number of 58¢ stamps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labels</td>
<td>Total cost = ( T ) (dollars) \hspace{1cm} Cost = 0.41 (dollars) \hspace{1cm} Number of 41¢ stamps = ( x ) (stamps) \hspace{1cm} Cost = 0.58 (dollars) \hspace{1cm} Number of 58¢ stamps = ( y ) (stamps)</td>
</tr>
<tr>
<td>Algebraic Model</td>
<td>( T = 0.41x + 0.58y )</td>
</tr>
</tbody>
</table>

\[
T = 0.41(12) + 0.58(10) \hspace{1cm} \text{Substitute 12 for } x \hspace{1cm} \text{and 10 for } y. \hspace{1cm} \text{Simplify.}
\]

\[
= 10.72
\]
1.6 Problem Solving Using Algebraic Models

**Goal**
- Use problem solving strategies to solve real-life problems.

**Example 1** Write and Use a Formula

**Travel** A bus travels at an average rate of 55 miles per hour. The distance between Chicago and San Francisco is 2130 miles. How many hours of driving will it take for the bus to travel from Chicago to San Francisco?

**Solution**

**Verbal Model**

Distance = Rate • Time

**Algebraic Model**

\[ 2130 = 55 \cdot t \]

An equation for this situation is \( 2130 = 55t \). Solve for \( t \).

\[ 2130 = 55t \quad \text{Write equation.} \]

\[ 38.7 \approx t \quad \text{Divide each side by 55.} \]

The driving time on the bus from Chicago to San Francisco is about 38.7 hours.

**Checkpoint** Complete the following exercise.

1. In Example 1, what is the average rate for the bus if it takes 22 hours to travel from San Francisco to Colorado Springs, a distance of 1335 miles?

   about 60.7 miles per hour
Travel The table gives the altitude \( a \) of a jet airplane \( t \) minutes after beginning its descent. Find the altitude of the airplane 9 minutes after it begins descending.

<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude, ( a ) (ft)</td>
<td>35,000</td>
<td>32,000</td>
<td>29,000</td>
<td>26,000</td>
<td>23,000</td>
</tr>
</tbody>
</table>

Solution

The altitude decreases by \( \frac{3000}{\text{feet per minute}} \).

\[
35,000 \quad 32,000 \quad 29,000 \quad 26,000 \quad 23,000
\]

\[
-3000 \quad -3000 \quad -3000 \quad -3000
\]

Use the pattern to write a verbal model for the altitude.

**Verbal Model**

\[
\text{Altitude} = \text{Initial altitude} - \text{Rate of descent} \times \text{Time}
\]

**Algebraic Model**

\[
a = 35,000 - \frac{3000}{t}
\]

An equation for the altitude is \( a = 35,000 - \frac{3000}{t} \).

So, the altitude 9 minutes after the plane begins descending is

\[
a = 35,000 - \frac{3000(9)}{9} = 35,000 - \frac{27,000}{9} = 8000 \text{ feet}.
\]

**Checkpoint** Complete the following exercise.

2. If a jet airplane descends at the rate indicated in the table below, what is its altitude 8 minutes after beginning its descent?

<table>
<thead>
<tr>
<th>Time, ( t ) (min)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude, ( a ) (ft)</td>
<td>36,000</td>
<td>32,800</td>
<td>29,600</td>
<td>26,400</td>
<td>23,200</td>
</tr>
</tbody>
</table>

10,400 feet
Home Improvement You want to paint five 1-foot wide vertical stripes on a wall. There is to be an equal amount of space between the ends of the wall and the stripes and between each pair of stripes. The wall is 14 feet long. How far apart should the stripes be?

Solution

Begin by drawing and labeling a diagram.

From the diagram, write and solve an equation to find $x$.

\[ \frac{6}{x} + \frac{5}{5} = 14 \]  
Write an equation.

\[ \frac{6x}{6} + 5 = 14 \]  
Simplify.

\[ 6x = 9 \]  
Subtract $5$ from each side.

\[ x = 1.5 \]  
Divide each side by $6$.

The stripes should be painted 1.5 feet apart.

Checkpoint Complete the following exercise.

3. In Example 3, how far apart would the stripes need to be if you were only going to paint 4 stripes on the wall?

2 feet apart
1.7 Analyzing and Displaying Data

**Goal**
- Use statistical measures and data displays to represent data.

**Vocabulary**

- **Mean**: In a data set of \( n \) numbers, the sum of the \( n \) numbers divided by \( n \).
- **Median**: The middle number of a data set when the numbers are written in numerical order.
- **Mode**: The number or numbers that occur most frequently in a data set.
- **Range**: The difference between the greatest and least data values.
- **Box-and-whisker plot**: A type of statistical graph in which a “box” encloses the middle half of the data set and “whiskers” extend to the minimum and maximum data values.
- **Lower quartile**: The median of the lower half of a data set.
- **Upper quartile**: The median of the upper half of a data set.

**Example 1**

*Find Measures of Central Tendency and Range*

Find the mean, median, mode(s), and range of these 14 quiz scores: 7, 16, 17, 19, 20, 20, 21, 22, 23, 24, 24, 24, 25, 25.

**Mean:**

\[
\text{Mean} = \frac{7 + 16 + 17 + 19 + 20 + 20 + 21 + 22 + 23 + 24 + 24 + 24 + 25 + 25}{14} = \frac{287}{14} = 20.5
\]

**Median:**

The median is between the 7th and 8th numbers, so the median is \( \frac{21 + 22}{2} = 21.5 \).

**Mode:**

The number that occurs most frequently is 24.

**Range:**

\( 25 - 7 = 18 \)

Remember that the numbers must be in order before finding the median. Here, the numbers are given in order.
Find the mean, median, mode, and range of these numbers of points scored by one team’s players in a basketball game: 24, 19, 15, 12, 12, 10, 8, 7, 4, 1.

mean: 11.2; median: 11; mode: 12; range: 23

Example 2  Find Lower and Upper Quartiles

Find the lower and upper quartile of the data in Example 1.

Solution

The values are already in order. The median is between 21 and 22. Each half of the data has 7 items.

Lower quartile: The median of 7, 16, 17, 19, 20, 20, and 21 is 19.

Upper quartile: The median of 22, 23, 24, 24, 24, 25, and 25 is 24.

Checkpoint  Complete the following exercise.

2. Find the lower and upper quartiles of the data in Checkpoint Exercise 1 above.

lower quartile: 7; upper quartile: 15
**Example 3**  **Draw a Box-and-Whisker Plot**

**Towers** The height, in feet, of the world’s ten tallest towers are listed below. Show how the heights are distributed by making a box-and-whisker plot.

\[
1214 \quad 1535 \quad 1230 \quad 1149 \quad 1815 \\
1369 \quad 1403 \quad 1362 \quad 1762 \quad 1198
\]

**Solution**

1. **Order** the data from least to greatest.

\[
1149, \underline{1198}, \underline{1214}, 1230, 1362, \underline{1369}, \\
1403, \underline{1535}, \underline{1762}, 1815
\]

2. **Find** the significant values for this data set.

- **The minimum** is **1149** and the **maximum** is **1815**.
- **median** \(\frac{1362 + 1369}{2} = 1365.5\)
- **lower half** of data: 1149, 1198, 1214, 1230, 1362
- **upper half** of data: 1369, 1403, 1535, 1762, 1815

3. **Plot** these five numbers below a **number line**.

4. **Draw** the box, the **whiskers**, and a **line segment** through the median.

![Box-and-Whisker Plot Diagram]

**Checkpoint**  Complete the following exercise.

3. **Draw a box-and-whisker plot for the data in Checkpoint Exercise 1.**
**1.8 Frequency Distributions and Histograms**

**Goal**
- Display data in frequency distributions and histograms.

**Your Notes**

**VOCABULARY**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Histogram</td>
<td>A special type of bar graph in which data values are grouped into intervals of the same size.</td>
</tr>
<tr>
<td>Frequency distribution</td>
<td>A table that shows how many times the numbers in each interval occur in the data</td>
</tr>
</tbody>
</table>

**Example 1  Make a Frequency Distribution**

**Weather** The average annual inches of precipitation in 20 United States cities is 16, 47, 54, 13, 44, 35, 31, 9, 48, 98, 50, 8, 36, 45, 66, 13, 51, 11, 21, 29. Make a frequency distribution of this data. Use five intervals beginning with the interval 1–20.

1. Write the five intervals.
   - The second interval extends from **21** to **40**.

2. Tally the data values by **interval**.

3. Count the tally marks to obtain the **frequencies**.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–20</td>
<td>•••••</td>
<td>6</td>
</tr>
<tr>
<td>21–40</td>
<td>•••</td>
<td>5</td>
</tr>
<tr>
<td>41–60</td>
<td>•••</td>
<td>7</td>
</tr>
<tr>
<td>61–80</td>
<td>•</td>
<td>1</td>
</tr>
<tr>
<td>81–100</td>
<td>•</td>
<td>1</td>
</tr>
</tbody>
</table>

Be sure your intervals have the same size.
**Your Notes**

**Example 2**  
*Draw a Histogram*

Draw a histogram for the rainfall data in Example 1.

1. Divide the horizontal axis into **five** equal sections. Label the sections with the **intervals** shown in the frequency distribution.

2. Draw a scale on the vertical axis to measure the **frequencies**.

3. Draw **bars** of the appropriate heights to represent the frequencies of the intervals. Label the **axes**, and include a title.

![Histogram of Average Annual Rainfall in 20 U.S. Cities]

**Checkpoint**  
Complete the following exercises.

A dog show has the following number of dogs of various breeds entered in competition: 2, 5, 17, 22, 15, 19, 12, 29, 30, 36, 52, 26, 34, 37, 37, 40, 45, 56, 50, 51.

1. Make a frequency distribution beginning with the interval 0–9.

<table>
<thead>
<tr>
<th>Interval</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–9</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>10–19</td>
<td>III</td>
<td>4</td>
</tr>
<tr>
<td>20–29</td>
<td>III</td>
<td>3</td>
</tr>
<tr>
<td>30–39</td>
<td>III</td>
<td>5</td>
</tr>
<tr>
<td>40–49</td>
<td>II</td>
<td>2</td>
</tr>
<tr>
<td>50–59</td>
<td>III</td>
<td>4</td>
</tr>
</tbody>
</table>

2. Draw a histogram.

![Histogram of Dogs Competing at a Dog Show]
# Words to Review

**Use your own words and/or an example to explain the vocabulary word.**

<table>
<thead>
<tr>
<th><strong>Origin</strong></th>
<th><strong>Graph</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The point labeled 0 on a real number line</td>
<td>The point on a number line that corresponds to a real number</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Coordinate</strong></th>
<th><strong>Opposite</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The number that corresponds to a point on a number line</td>
<td>The opposite of 5 is $-5$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Reciprocal</strong></th>
<th><strong>Base</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The reciprocal of 5 is $\frac{1}{5}$.</td>
<td>In $2^3$, the base is 2.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Exponent</strong></th>
<th><strong>Power</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>In $2^3$, the exponent is 3.</td>
<td>$2^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Numerical expression</strong></th>
<th><strong>Variable</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$10 + (4 + 3^2)$</td>
<td>In $2x + 3$, the variable is $x$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Algebraic expression</strong></th>
<th><strong>Term</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x - 2y + 11$</td>
<td>$2x$ is a term in $5 + 2x$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Coefficient</strong></th>
<th><strong>Like terms</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The coefficient of $x$ in $2x$ is 2.</td>
<td>$2x$ and $7x$ are like terms.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Constant term</strong></th>
<th><strong>Simplified expression</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>7 is the constant term in $2x + 7$.</td>
<td>An expression in which all grouping symbols have been removed and all like terms have been combined</td>
</tr>
<tr>
<td>Equation</td>
<td>Linear equation</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$2x + 3 = 7$</td>
<td>$5x - 20 = 0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solution</th>
<th>Verbal model</th>
</tr>
</thead>
<tbody>
<tr>
<td>The solution of $5x = 10$ is 2.</td>
<td>An equation written in words</td>
</tr>
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</table>

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<thead>
<tr>
<th>Algebraic model</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>A mathematical statement for a real-life situation</td>
<td>In a data set of $n$ numbers, the sum of the $n$ numbers divided by $n$.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Median</th>
<th>Mode</th>
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</thead>
<tbody>
<tr>
<td>The middle number of a data set when the numbers are written in numerical order</td>
<td>The number or numbers that occur most frequently in a data set</td>
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<table>
<thead>
<tr>
<th>Range</th>
<th>Box-and-whisker plot</th>
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<tr>
<td>The difference between the greatest and least data values</td>
<td>A type of statistical graph in which a “box” encloses the middle half of the data set and “whiskers” extend to the minimum and maximum data values</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower quartile</th>
<th>Upper quartile</th>
</tr>
</thead>
<tbody>
<tr>
<td>The median of the lower half of a data set</td>
<td>The median of the upper half of a data set</td>
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Review your notes and Chapter 1 by using the Chapter Review on pages 57–60 of your textbook.